

## 9-2 廣義角與極座標(常考題型 1)

下列哪些是 $-93^\circ$ 的同界角?

(1) $93^\circ$  (2) $267^\circ$  (3) $357^\circ$  (4) $-453^\circ$  (5) $467^\circ$ .



解答

24

解析

$-93^\circ$ 的同界角均可表為 $(-93^\circ) + 360^\circ \times n$ ,  $n$ 為整數.

$267^\circ = (-93^\circ) + 360^\circ$ ,  $-453^\circ = (-93^\circ) + 360^\circ \times (-1)$ ,  
故選(2)(4).

## 9-2 廣義角與極座標(常考題型 2)

已知 $P(-3, 4)$ 為標準位置角 $\theta$ 終邊上的一點,

求(1) $\sin \theta =$ \_\_\_\_\_ . (2) $\cos \theta =$ \_\_\_\_\_ . (3) $\tan \theta =$ \_\_\_\_\_

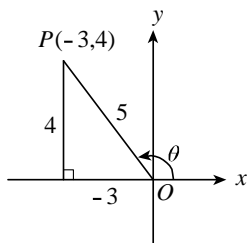


解答

(1) $\frac{4}{5}$ ; (2) $-\frac{3}{5}$ ; (3) $-\frac{4}{3}$

解析

$\because P(-3, 4)$ ,  $x = -3$ ,  $y = 4 \Rightarrow r = \overline{OP} = \sqrt{(-3)^2 + 4^2} = 5$



$\therefore$  (1) $\sin \theta = \frac{y}{r} = \frac{4}{5}$  . (2) $\cos \theta = \frac{x}{r} = -\frac{3}{5}$  . (3) $\tan \theta = \frac{y}{x} = -\frac{4}{3}$  .

## 9-2 廣義角與極座標(常考題型 3)

設  $\sin \theta = \frac{1}{3}$ ,  $90^\circ < \theta < 180^\circ$ , 試求下列各三角函數值:

(1)  $\cos \theta$  .

(2)  $\tan \theta$  .

(3)  $\sin(90^\circ + \theta)$  .

(4)  $\cos(180^\circ - \theta)$  .

(5)  $\sin(270^\circ + \theta)$  .

(6)  $\tan(450^\circ + \theta)$  .

(7)  $\cos(630^\circ - \theta)$  .

(8)  $\sin(-630^\circ + \theta)$  .



解答

(1)  $-\frac{2\sqrt{2}}{3}$ ; (2)  $-\frac{1}{2\sqrt{2}}$ ; (3)  $-\frac{2\sqrt{2}}{3}$ ; (4)  $\frac{2\sqrt{2}}{3}$ ; (5)  $\frac{2\sqrt{2}}{3}$ ; (6)  $2\sqrt{2}$ ; (7)  $-\frac{1}{3}$ ; (8)  $-\frac{2\sqrt{2}}{3}$

解析

(1)  $\cos \theta = -\frac{2\sqrt{2}}{3}$  .

(2)  $\tan \theta = -\frac{1}{2\sqrt{2}}$  .

(3)  $\sin(90^\circ + \theta) = \cos \theta = -\frac{2\sqrt{2}}{3}$  .

(4)  $\cos(180^\circ - \theta) = -\cos \theta = \frac{2\sqrt{2}}{3}$  .

(5)  $\sin(270^\circ + \theta) = -\cos \theta = \frac{2\sqrt{2}}{3}$  .

(6)  $\tan(450^\circ + \theta) = -\cot \theta = -(-2\sqrt{2}) = 2\sqrt{2}$  .

(7)  $\cos(630^\circ - \theta) = -\sin \theta = -\frac{1}{3}$  .

(8)  $\sin(-630^\circ + \theta) = \sin[-(630^\circ - \theta)] = -\sin(630^\circ - \theta) = -(-\cos \theta)$   
 $= \cos \theta = -\frac{2\sqrt{2}}{3}$  .

## 9-2 廣義角與極座標(常考題型 4)

化簡  $\sin(180^\circ + \theta) \cdot \cos(90^\circ + \theta) - \sin(90^\circ - \theta) \cdot \cos(180^\circ - \theta) =$

\_\_\_\_\_ .



解答

1

解析

原式  $= (-\sin\theta)(-\sin\theta) - \cos\theta \cdot (-\cos\theta) = \sin^2\theta + \cos^2\theta = 1$  .

## 9-2 廣義角與極座標(常考題型 5)

求  $\cos 225^\circ \cdot \cos 315^\circ - \sin 210^\circ \cdot \cos 240^\circ + \tan 225^\circ$  .



解答

$\frac{1}{4}$

解析

原式  $= \cos(180^\circ + 45^\circ) \cdot \cos(360^\circ - 45^\circ) - \sin(180^\circ + 30^\circ) \cdot \cos(180^\circ + 60^\circ) - \tan(180^\circ + 45^\circ)$   
 $= (-\cos 45^\circ)(\cos 45^\circ) - (-\sin 30^\circ) \cdot (-\cos 60^\circ) + \tan 45^\circ$   
 $= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + 1 = -\frac{1}{2} - \frac{1}{4} + 1 = \frac{1}{4}$  .

## 9-2 廣義角與極座標(常考題型 6)

設  $\sin(-123^\circ) = k$ , 試以  $k$  表示  $\cos 237^\circ = ?$



解答

$$-\sqrt{1-k^2}$$

解析

$$\sin(-123^\circ) = k \Rightarrow -\sin 123^\circ = k \Rightarrow \sin 123^\circ = -k$$

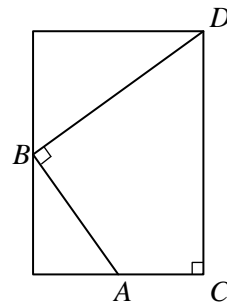
$$\sin(180^\circ - 57^\circ) = -k \Rightarrow \sin 57^\circ = -k$$

$$\cos 237^\circ = \cos(180^\circ + 57^\circ) = -\cos 57^\circ = -\sqrt{1 - \sin^2 57^\circ} = -\sqrt{1 - (-k)^2} = -\sqrt{1 - k^2} .$$

## 9-2 廣義角與極座標(常考題型 7)

如圖,  $\angle BAC = \theta$ ,  $\angle ABD = \angle ACD = 90^\circ$ ,  $\overline{AB} = a$ ,  $\overline{BD} = b$ . 下列選項何者可以表示  $\overline{CD}$ ?

- (1)  $a \sin \theta + b \cos \theta$
- (2)  $a \sin \theta - b \cos \theta$
- (3)  $a \cos \theta - b \sin \theta$
- (4)  $a \cos \theta + b \sin \theta$
- (5)  $a \sin \theta + b \tan \theta$



解答

2

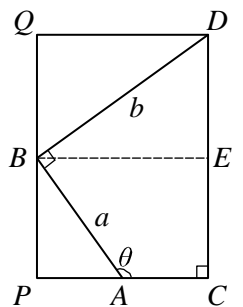
解析

過  $B$  作  $\overline{BE} \perp \overline{CD}$  於  $E$  點, 作  $\overline{BP} \perp \overline{CA}$  於  $P$  點  $\Rightarrow \angle BAP = 180^\circ - \theta = \angle BDE$

$$\overline{CD} = \overline{CE} + \overline{ED} = \overline{BP} + \overline{ED} = \overline{AB} \sin(180^\circ - \theta) + \overline{BD} \cos(180^\circ - \theta)$$

$$= a \sin(180^\circ - \theta) + b \cos(180^\circ - \theta) = a \sin \theta - b \cos \theta$$

故選(2) .



## 9-2 廣義角與極座標(常考題型 8)

求  $\sum_{k=1}^{179} \cos^2 k^\circ$  的值為\_\_\_\_\_。



解答

89

解析

$$\sum_{k=1}^{179} \cos^2 k^\circ = \cos^2 1^\circ + \cos^2 2^\circ + \cos^2 3^\circ + \cdots + \cos^2 91^\circ + \cdots + \cos^2 179^\circ$$

$$\because \cos^2 91^\circ = \cos^2(90^\circ + 1^\circ) = \sin^2 1^\circ$$

$$\therefore \cos^2 91^\circ + \cos^2 1^\circ = \sin^2 1^\circ + \cos^2 1^\circ = 1$$

$$\text{同理 } \cos^2 92^\circ + \cos^2 2^\circ = 1$$

⋮

$$\cos^2 179^\circ + \cos^2 89^\circ = 1 \quad \therefore \text{原式} = 89.$$